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VEDIC MATHEMATICS
OR
‘SIXTEEN SIMPLE MATHEMATICAL
FORMULAE FROM THE VEDAS’

BY
JAGADGURU
SWĀMĪ ŚRĪ BHĀRATĪ KṚṢṆA TĪRTHAJĪ MAHĀRĀJA,
SHANKARACHARYA OF GOVARDHANA MATHA, PURI.

BANARAS HINDU UNIVERSITY

1965.

॥ ॐ श्रीः ॥



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VEDIC MATHEMATICS

OR

‘SIXTEEN SIMPLE MATHEMATICAL FORMULAE FROM THE VEDAS’

(For One-line Answers to All Mathematical Problems)

BY

JAGADGURU

SWĀMI ŚRĪ BHĀRATĪ KṚṢṆA TĪRTHAJĪ MAHĀRĀJA,
SHANKARACHARYA OF GOVARDHANA MATHA, PURI.



General Editor

Dr. V. S. AGRAWALA

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PUBLICATION ANNOUNCEMENT

I have great pleasure in associating myself with the publication of the book Vedic Mathematics or 'Sixteen Simple Mathematical Formulae,' by Jagadguru Swami Bharati Krishna Tirtha, Shankaracharya of Govardhana Pitha. It was long awaited by his disciples. Shrimati Manjula Devi, sole inheriter of Śwamiji's right, entered into an agreement with the Banaras Hindu University to publish it and the same is now being done in the Nepal Endowment Hindu Vishvavidyalaya Sanskrit Granthamala.

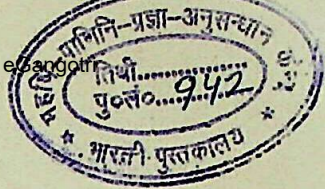
I feel grateful to all those who have worked for it. Shri Arvind N. Mafatlal business magnate of Bombay and a devotee of Swamiji has taken interest in the publication of the work. He has taken the trouble of being personally present in this ceremony of publication (*Prakashana Utsava*). He has given expression to his deep devotion to Shri Shankaracharyaji by consenting to found a chair at the Banaras Hindu University by the name of Shri Jagadguru Bharati Krishna Tirtha Shankaracharya Chair of Vedic Studies for which he is making a magnificent endowment. As Vice-Chancellor of this University I accept the donation and offer my heart-felt thanks to him for his generosity.

N. H. BHAGWATI

Vice-Chancellor

Banaras Hindu University

Date 27-3-65



GENERAL EDITOR'S FOREWORD

The work entitled VEDIC MATHEMATICS or 'Sixteen Simple Mathematical Formulæ from the Vedas' was written by His Holiness Jagadguru Śaṅkarācārya Śrī Bhāratī Kṛṣṇa Tīrthajī Mahārāja of Govardhana Maṭha, Puri (1884-1960). It forms a class by itself not pragmatically conceived and worked out as in the case of other scientific works, but the result of the intuitional visualisation of fundamental mathematical truths and principles during the course of eight years of highly concentrated mental endeavour on the part of the author and therefore appropriately given the title of "mental" mathematics appearing more as miracle than the usual approach of hard-baked science, as the author has himself stated in the Preface.

Swāmī Śaṅkarācārya was a gifted scholar on many fronts of learning including science and humanities but his whole milieu was something of a much higher texture viz, that he was a Ṛṣi fulfilling the ideals and attainments of those Seers of ancient India who discovered the cosmic laws embodied in the Vedas. Swāmī Bhāratī Kṛṣṇa Tīrtha had the same reverential approach towards the Vedas. The question naturally arises as to whether the Sūtras which form the basis of this treatise exist anywhere in the Vedic literature as known to us. But this criticism loses all its force if we inform ourselves of the definition of Veda given by Śrī Śaṅkarācārya himself as quoted below :

"The very word 'Veda' has this derivational meaning i.e. the fountain-head and illimitable store-house of all knowledge. This derivation, in effect, means, connotes and implies that the Vedas *should contain* (italics mine) within themselves all the knowledge needed by mankind relating not only to the so-called 'spiritual' (or other-worldly) matters but also to those usually described as purely 'secular', 'temporal', or 'worldly' and also to the means required by humanity as such for the achievement of all-round, complete and perfect success in all conceivable directions and that there can be no adjectival or restrictive epithet calculated (or tending) to limit that knowledge down in any sphere, any direction or any respect whatsoever.

"In other words, it connotes and implies that our ancient Indian Vedic lore *should be* (italics mine) all-round, complete and perfect and able to throw the fullest necessary light on all matters which any aspiring seeker after knowledge can possibly seek to be enlightened on".

It is the whole essence of his assessment of Vedic tradition that it is not to be approached from a factual standpoint but from the ideal standpoint viz, as the Vedas as traditionally accepted in India as the repository of all knowledge *should be* and not what they are in human possession. That approach entirely turns the tables on all critics, for the authorship of Vedic mathematics then need not be laboriously searched in the texts as preserved from antiquity. The Vedas are well known as four in number R̥k, Yaju, Sāma and Atharva but they have also the four Upavedas and the six Vedāṅgas all of which form an indivisible corpus of divine knowledge as it once was and as it may be revealed. The four Upavedas are as follows :—

<i>Veda</i>	<i>Upaveda</i>
R̥gveda	Āyurveda
Sāmaveda	Gāndharvaveda
Yajurveda	Dhanurveda
Atharvaveda	Sthāpathyaveda

In this list the Upaveda of Sthāpatya or engineering comprises all kinds of architectural and structural human endeavour and all visual arts. Swamīji naturally regarded mathematics or the science of calculations and computations to fall under this category.

In the light of the above definition and approach must be understood the author's statement that the sixteen Sūtras on which the present volume is based form part of a *Parīṣiṣṭa* of the Atharvaveda. We are aware that each Veda has its subsidiary apocryphal texts some of which remain in manuscripts and others have been printed but that formulation has not closed. For example, some *Parīṣiṣṭas* of the Atharvaveda were edited by G. M. Bolling and J. Von Negelein, Liepzing, 1909-10. But this work of Śrī Śaṅkarācāryaji deserves to be regarded as a new *Parīṣiṣṭa* by itself and it is not surprising that the Sūtras

mentioned herein do not appear in the hitherto known *Parīṣ-īṣṭas*.

A list of these main 16 *Sūtras* and of their sub-*sūtras* or corollaries is prefixed in the beginning of the text and the style of language also points to their discovery by Śrī Swāmījī himself. At any rate, it is needless to dwell longer on this point of origin since the vast merit of these rules should be a matter of discovery for each intelligent reader. Whatever is written here by the author stands on its own merits and is presented as such to the mathematical world.

Swāmījī was a marvellous person with surpassing qualities and was a prolific writer and eloquent speaker. I had the good fortune of listening to his discourses for weeks together on several occasions when he used to visit Lucknow and attracted large audiences. He could at a stretch speak for several hours in Sanskrit and English with the same facility and the intonation of his musical voice left a lasting impression on the minds of his hearers. He was an ardent admirer of Bhartṛhari the great scientific thinker of the Golden Age of Indian history in a different field viz, that of philosophy of grammar.

Swāmījī had planned to write 16 volumes on all aspects and branches of mathematical processes and problems and there is no doubt that his mental powers were certainly of that calibre, but what has been left to us is this introductory volume which in itself is of the highest merit for reason of presenting a new technique which the author styles as “mental” mathematics different from the orthodox methods of mathematicians all over the world. Arithmetical problems usually solved by 18, 28 or 42 steps in case of such vulgar fractions as $1/19$, $1/29$, $1/49$ are here solved in one simple line and that is possible to be done even by young boys. The truth of these methods was demonstrated by this saintly teacher before many University audiences in India and in the U.S.A. including learned Professors and every one present was struck with their originality and simplicity.

We are told in his Preface by Swāmī Śaṅkarācārya that he contemplated to cover all the different branches of mathe-

matics such as arithmetic, algebra, geometry (plane and solid) trigonometry (plane and spherical) conics—geometrical and analytical, astronomy, calculus—differential and integral etc., with these basic Sūtras. That comprehensive application of the Sūtras could not be left by him in writing but if some one has the patience and the genius to pursue the method and implications of these formulae he may probably be able to bring these various branches within the orbit of this original style.

A full fledged course of his lecture-demonstrations was organised by the Nagpur University in 1952 and some lectures were delivered by Swamiji at the B.H.U. in 1949. It is, therefore, in the fitness of things and a happy event for the B.H.U. to be given the opportunity of publishing this book by the courtesy of Srimati Manjula Devi Trivedi, disciple of Śrī Swāmījī who agreed to make over this manuscript to us through the efforts of Dr. Pt. Omkarnath Thakur. The work has been seen through the Press mainly by Dr. Prem Lata Sharma, Dean, Faculty of Music & Fine Arts in the University. To all of these our grateful thanks are due. Dr. Brij Mohan, Head of the Department of Mathematics, B.H.U., took the trouble, at my request, of going through the manuscript and verifying the calculations for which I offer him my best thanks. I also express gratitude to Sri Lakshmidas, Manager, B.H.U. Press, for taking great pains in printing this difficult text.

We wish to express our deepest gratitude to Śrī Swāmī Pratyagātmānanda Saraswatī for the valuable foreword that he has written for this work. Today he stands pre-eminent in the world of Tantric scholars and is a profound mathematician and scientific thinker himself. His inspiring words are like fragrant flowers offered at the feet of the ancient Vedic Rsis whose spiritual lineage was revealed in the late Śankarācārya Śrī Bhārati Kṛṣṇa Tīrtha. Swāmī Pratyagātmānandajī has not only paid a tribute to Śrī Śankarācāryajī but his ambrosial words have showered blessings on all those who are lovers of intuitional experiences in the domain of metaphysics and physics. Swamiji, by a fortunate chance, travelled from Calcutta

to Varanasi to preside over the Tantric Sammelan of the Varanaseya Sanskrit University (8th to 11th March 1965) and although he is now 85 years of age, his innate generosity made him accept our request to give his foreword.

I am particularly happy that I am able to publish this work under the Nepal Endowment Hindu Vishwavidyalaya Publication Series, for I entertained an ardent desire to do so since our late President Dr. Rajendra Prasadji spoke to me about its existence when I once met him in New Delhi in the lifetime of Śrī Swāmījī.

Banaras Hindu University
Varanasi-5
March 17, 1965.

V. S. AGRAWALA,
M.A., Ph.D., D.Litt.
General Editor,
Hindu Vishwavidyalaya
Nepal Rajya Sanskrit
Granthamala Series.

FOREWORD .

Vedic Mathematics by the late Śaṅkarācārya (Bhārati Kṛṣṇa Tirtha) of Govardhana Pīṭha is a monumental work. In his deep-layer explorations of cryptic Vedic mysteries relating specially to their calculus of shorthand formulae and their neat and ready application to practical problems, the late Śaṅkarācārya shews the rare combination of the probing insight and revealing intuition of a Yogī with the analytic acumen and synthetic talent of a mathematician. With the late Śaṅkarācārya we belong to a race, now fast becoming extinct, of die-hard believers who think that the Vedas represent an inexhaustible mine of profoundest wisdom in matters both spiritual and temporal ; and that this store of wisdom was not, as regards its assets of fundamental validity and value at least, gathered by the laborious inductive and deductive methods of ordinary systematic enquiry, but was a direct gift of revelation to seers and sages who in their higher reaches of Yogic realization were competent to receive it from a Source, perfect and immaculate. But we admit, and the late Śaṅkarācārya has also practically admitted, that one cannot expect to convert or revert criticism, much less carry conviction, by merely asserting one's staunchest beliefs. To meet these ends, one must be prepared to go the whole length of testing and verification by accepted, accredited methods. The late Śaṅkarācārya has, by his comparative and critical study of Vedic mathematics, made this essential requirement in Vedic studies abundantly clear. So let us agree to gauge Vedic mysteries not as we gauge the far-off nabulae with the poet's eye or with that of the seer, but with the alert, expert, scrutinizing eye of the physical astronomer, if we may put it as that.

That there is a consolidated metaphysical background in the Vedas of the objective sciences including mathematics as regards their basic conceptions is a point that may be granted by a thinker who has looked broadly and deeply into both the realms.

In our paper recently published—'The Metaphysics of Physics'—we attempted to look into the mysteries of creative emergence as contained in the well-known cosmogenic Hymn

(Rg. X.190) with a view to unveiling the metaphysical background where both ancient wisdom and modern physics may meet on a common basis of logical understanding, and compare notes, discovering, where possible, points of significant or suggestive parallelism between the two sets of concepts, ancient and modern. That metaphysical background includes mathematics also; because physics as ever pursued is the application of mathematics to given or specified space-time-event situations. There we examined Tapas as a fundamental creative formula whereby the Absolute emerges into the realms of measures, variations, limits, frame-works and relations. And this descent follows a logical order which seems to lend itself, within a framework of conditions and specifications, to mathematical analysis. *Rātri* in the Hymn represents the Principle of Limits, for example, *Rtañca Satyañca* stand for Becoming (*Calana-kalana*) and Being (*varṭana-kalana*) at a stage where limits or conditions or conventions do not yet arise or apply. The former gives the unconditioned, unrestricted *how* or *thus* of cosmic process; the latter, *what* or *that* of existence. *Tapas*, which corresponds to *Ardhamātrā* in Tantric symbolism, negotiates, in its role specially of critical variation, between what is, *ab-initio*, unconditioned and unrestricted, and what appears otherwise, as for instance, in our own universe of logico-mathematical appreciation.

This is, necessarily, abstruse metaphysics, but it is, nevertheless, the starting background of both physics and mathematics. But for all practical purposes we must come down from mystic nabulae to the *terra firma* of our actual apprehension and appreciation. That is to say, we must descend to our own pragmatic levels of time-space-event situations. *Here* we face actual problems, and one must meet and deal with these squarely without evasion or mystification. The late Śaṅkarācārya has done this masterly feat with an adroitness that compels admiration.

It follows from the fundamental premises that the universe we live in must have a basic mathematical structure, and consequently, to know a fact or obtain a result herein, to any required degree of precision, one must obey the rule of mathe-

mathematical measures and relations. This, however, one may do consciously or semi-consciously, systematically or haphazardly. Even some species of lower animals are by instinct gifted mathematicians ; for example, the migratory bird which flies thousands of miles off from its nest-home, and after a period, unerringly returns. This implies a subconscious mathematical talent that works wonder. We may cite the case of a horse who was a mathematical prodigy and could 'tell' the result of a cube root (requiring 32 operations, according to M. Materlink in his 'Unknown Quest') in a twinkling of the eye. This sounds like magic, but it is undeniable that the feat of mathematics *does* sometimes assume a magical look. Man, undoubtedly, has been given his share of this magical gift. And he can improve upon it by practice and discipline, by *Yoga* and allied methods. This is undeniable also. Lately, he has devised the 'automatic brain' for complicated calculations by science, that looks like magic.

But apart from this 'magic', there is and has been, the 'logic' of mathematics also. Man works from instinct, talent, or even genius. But ordinarily he works as a logical entity requiring specified data or premises to start from, and more or less elaborate steps of reasoning to arrive at a conclusion. This is his normal process of induction and deduction. Here formulæ (*Sūtras*) and relations (e.g. equations) must obtain as in mathematics. The magic and logic of mathematics in some cases get mixed up ; but it is sane to keep them apart. You can get a result by magic, but when you are called upon to *prove*, you must have recourse to logic.

Even in this latter case, your logic (your formulæ and applications) may be either simple and elegant or complicated and cumbrous. The former is the ideal to aim at. We have classical instances of master mathematicians whose methods of analysis and solution have been regarded as marvels of cogency, compactness and elegance. Some have been 'beautiful' as a poem (e.g. Lagrange's 'Analytical Mechanics.')

The late Śaṅkarācārya has claimed, and rightly we may think, that the Vedic *Sūtras* and their applications possess these

virtues to a degree of eminence that cannot be challenged. The outstanding merit of his work lies in his actual proving of this contention.

Whether or not the Vedas be believed as repositories of perfect wisdom, it is unquestionable that the Vedic race lived *not* as merely pastoral folk possessing a half-or-quarter-developed culture and civilization. The Vedic seers were, again, not mere 'navel-gazers' or 'nose-tip-gazers'. They proved themselves adepts in all levels and branches of knowledge, theoretical and practical. For example, they had their varied objective science, both pure and applied.

Let us take a concrete illustration. Suppose in a time of drought we require rains by artificial means. The modern scientist has his own theory and art (technique) for producing the result. The old seer scientist had his both also, but different from these now availing. He had his science and technique, called *Yajña*, in which *Mantra*, *Yantra* and other factors must co-operate with mathematical determinateness and precision. For this purpose, he had developed the six auxiliaries of the Vedas in each of which mathematical skill and adroitness, occult or otherwise, play the decisive role. The *Sūtras* lay down the shortest and surest lines. The correct intonation of the *Mantra*, the correct configuration of the *Yantra* (in the making of the *Vedī* etc., e.g. the quadrature of a circle), the correct time or astral conjugation factor, the correct rhythms etc., all had to be perfected so as to produce the desired result effectively and adequately. Each of these required the calculus of mathematics. The modern technician has his logarithmic tables and mechanics' manuals; the old Yājñika had his *Sūtras*. How were the *Sūtras* obtained?—by magic or logic or both?—is a vital matter we do not discuss here. The late Śaṅkarācārya has claimed for them cogency, compactness and simplicity. This is an even more vital point, and we think, he has reasonably made it good.

Varanasi,
22-3-1965

SWĀMĪ PRATYAGĀTMĀNANDA
SARASWATĪ

A HUMBLE HOMAGE

The late Śaṅkarācārya's epoch-making work on Vedic-Mathematics brings to the notice of the intelligentsia most strikingly a new theory and method, now almost unknown, of arriving at the truth of things which in this particular case concerns the truth of numbers and magnitude, but might as well cover, as it undoubtedly did in a past age in India, all sciences and arts, with results which do not fail to evoke a sense of awe and amazement today. The method obviously is radically different from the one adopted by the modern mind.

Music and not Mathematics is my field (although the philosophy of numbers, cosmic and metaphysical correspondences with musical numbers, the relation of numbers with consonant, dissonant and assonant tonal intervals etc., closely inter-relate music and mathematics), but study of the traditional literature on music and fine arts with which I have been concerned for the last few years has convinced me of one fundamental fact regarding the ancient Indian theory and method of knowledge and experience *vis a vis* the modern. While all great and true knowledge is born of intuition and not of any rational process or imagination, there is a radical difference between the ancient Indian method and the modern Western method concerning intuition.

The divergence embraces everything other than the fact of intuition itself—the object and field of intuitive vision, the method of working out experience and rendering it to the intellect. The modern method is to get the intuition by suggestion from an appearance in life or nature or from a mental idea and even if the source of the intuition is the soul, the method at once relates it to a support external to the soul. The ancient Indian method of knowledge had for its business to disclose something of the Self, the Infinite or the Divine to the regard of the soul—the Self through its expressions, the infinite through its finite symbols and the Divine through his powers. The

process was one of Integral knowledge and in its subordinate ranges was instrumental in revealing the truths of cosmic phenomena and these truths were utilised for worldly ends.

These two methods are based on different theories of knowledge and experience, fundamentally divergent in outlook and approach. The world as yet knows very little of the ancient Indian method, much less of its secret techniques. Śrī Śaṅkarācārya's remarkably unique work of Vedic mathematics has brought to popular notice demonstrably for the first time that the said method was usefully employed in ancient India in solving problems of secular knowledge just as for solving those of the spiritual domain.

I am happy that in the printing and publication of this monumental work and the preceding spade-work I had the privilege to render some little service.

Varanasi-5.
23-3-65.

PREM LATA SHARMA
Dean, Faculty of Music & Fine Arts,
Banaras Hindu University.

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MY BELOVED GURUDEVA

SMTI. MANJULA TRIVEDI

[In the lines that follow the writer gives a short biographical sketch of the illustrious author of Vedic Mathematics and a short account of the genesis of his work now published, based on intimate personal knowledge—EDITOR.]

Very few persons can there be amongst the cultured people of India who have not heard about HIS HOLINESS JAGAD-GURU SHANKARACHARYA SRI BHARATI KRISHNA TIRTHAJI MAHARAJ, the magnificent and divine personality that gracefully adorned the famous Govardhan Math, Puri, his vast and versatile learning, his spiritual and educational attainments, his wonderful research achievements in the field of Vedic Mathematics and his consecration of all these qualifications to the service of humanity as such.

His Holiness, better known among his disciples by the beloved name 'Jagadguruji' or 'Gurudeva' was born of highly learned and pious parents in March, 1884. His father, late Sri P. Narasimha Shastri, was then in service as a Tahsildar at Tinnivelly (Madras Presidency) who later retired as a Deputy Collector. His uncle, late Shri Chandrashekhar Shastri, was the Principal of the Maharaja's College, Vizianagaram and his great-grandfather was late Justice C. Ranganath Shastri of the Madras High Court.

Jagadguruji, named as Venkatraman in his early days, was an exceptionally brilliant student and invariably won the first place in all the subjects in all the classes throughout his educational career. During his school days, he was a student of National College, Trichanapalli; Church Missionary Society College, Tinneveli and Hindu College, Tinneveli. He passed his matriculation examination from the Madras University in January, 1899, topping the list as usual.

He was extra-ordinarily proficient in Sanskrit and oratory and on account of this he was awarded the title of 'SARASWATI'

by the Madras Sanskrit Association in July, 1899 when he was still in his 16th year. One cannot fail to mention at this stage the profound impression left on him by his Sanskrit Guru Shri Vedam Venkatrai Shastri whom Jagadguruji always remembered with deepest love, reverence and gratitude, with tears in his eyes.

After winning the highest place in the B.A. Examination, Shri Venkatraman Saraswati appeared at the M.A. Examination of the American College of Sciences, Rochester, New York, from Bombay Centre in 1903 ; and in 1904 at the age of just twenty he passed M.A. Examination in further seven subjects simultaneously securing the highest honours in all, which is perhaps the all-time world-record of academic brilliance. His subjects included Sanskrit, Philosophy, English, Mathematics, History and Science.

As a student Venkatraman was marked for his splendid brilliance, superb retentive memory and ever-insatiable curiosity. He would deluge his teachers with myriads of piercing questions which made them uneasy and forced them frequently to make a frank confession of ignorance on their part. In this respect, he was considered to be a terribly mischievous student.

Even from his University days Shri Venkatraman Saraswati had started contributing learned articles on religion, philosophy, sociology, history, politics, literature etc., to late W. T. Stead's "REVIEW OF REVIEWS" and he was specially interested in all the branches of modern science. In fact, study of the latest researches and discoveries in modern science continued to be Shri Jagadguruji's hobby till his very last days.

Sri Venkatraman started his public life under the guidance of late Hon'ble Shri Gopal Krishna Gokhale, C.I.E. in 1905 in connection with the National Education Movement and the South African Indian issue. Although, however, on the one hand, Prof. Venkatraman Saraswati had acquired an endless fund of learning and his desire to learn ever more was still unquenchable and on the other hand the urge for selfless service

of humanity swayed his heart mightily, yet the undoubtedly deepest attraction that Venkatraman Sarāswati felt was that towards the study and practice of the science of sciences—the holy ancient Indian spiritual science or Adhyātma-Vidyā. In 1908, therefore, he proceeded to the Sringeri Math in Mysore to lay himself at the feet of the renowned late Jagadguru Shankaracharya Maharaj Shri Satchidānanda Sivābhinava Nrisimha Bhārati Swami.

But he had not stayed there long, before he had to assume the post of the first Principal of the newly started National College at Rajmahendri under a pressing and clamant call of duty from the nationalist leaders. Prof. Venkatraman Saraswati continued there for three years but in 1911 he could not resist his burning desire for spiritual knowledge, practice and attainment any more and, therefore, tearing himself off suddenly from the said college he went back to Shri Satchidānanda Sivābhinava Nrisimha Bhārati Swami at Sringeri.

The next eight years he spent in the profoundest study of the most advanced Vedānta Philosophy and practice of the Brahma-sādhana. During these days Prof. Venkatraman used to study Vedānta at the feet of Shri Nrisimha Bhārati Swami, teach Sanskrit and Philosophy in schools there, and practise the highest and most vigorous Yoga-sādhāna in the nearby forests. Frequently, he was also invited by several institutions to deliver lectures on philosophy; for example he delivered a series of sixteen lectures on Shankaracharya's Philosophy at Shankar Institute of Philosophy, Amalner (Khandesh) and similar lectures at several other places like Poona, Bombay etc.

After several years of the most advanced studies, the deepest meditation, and the highest spiritual attainment Prof. Venkatraman Saraswati was initiated into the holy order of SAMNYASA at Banaras (Varanasi) by his Holiness Jagadguru Shankaracharya Sri Trivikram Tirthaji Maharaj of Shārādāpeeth on the 4th July 1919 and on this occasion he was given the new name, Swami Bharati Krishna Tirtha.

This was the starting point of an effulgent manifestation of Swamiji's real greatness. Within two years of his stay in the holy order, he proved his unique suitability for being installed on the pontifical throne of Sharada Peetha Shankaracharya and accordingly in 1921, he was so installed with all the formal ceremonies despite all his reluctance and active resistance. Immediately, on assuming the pontificate Shri Jagadguruji started touring India from corner to corner and delivering lectures on Sanātana Dharma and by his scintillating intellectual brilliance, powerful oratory, magnetic personality, sincerity of purpose, indomitable will, purity of thought, and loftiness of character he took the entire intellectual and religious class of the nation by storm.

Jagadguru Shankaracharya Shri Madhusudan Tirtha of Govardhan Math Puri was at this stage greatly impressed by Jagadguruji and when the former was in failing health he requested Jagadguruji to succeed him on Govardhan Math Gadi. Shri Jagadguruji continued to resist his importunate requests for a long time but at last when Jagadguru Shri Madhusudan Tirtha's health took a serious turn in 1925 he virtually forced Jagadguru Shri Bharati Krishana Tirthaji to accept the Govardhan Math's Gadi and accordingly Jagadguruji installed Shri Swarupanandji on the Sharadapeeth Gadi and himself assumed the duties of the ecclesiastical and pontifical head of Sri Govardhan Math, Puri.

In this capacity of Jagadguru Shankaracharya of Govardhan Math, Puri, he continued to disseminate the holy spiritual teachings of Sanatana Dharma in their pristine purity all over the world the rest of his life for 35 years. Months after months and years after years he spent in teaching and preaching, talking and lecturing, discussing and convincing millions of people all over the country. He took upon himself the colossal task of the renaissance of Indian culture, spreading of Sanatana Dharma, revival of the highest human and moral values and enkindling of the loftiest spiritual enlightenment throughout the world and he dedicated his whole life to this lofty and noble mission.

From his very early days Jagadguruji was aware of the need for the right interpretation of "Dharma" which he defined as "the sum total of all the means necessary for speedily making and permanently keeping all the people, individually as well as collectively superlatively comfortable, prosperous, happy, and joyous in all respects (including the physical, mental, intellectual, educational, economic, social, political, psychic, spiritual etc. *ad infinitum*)". He was painfully aware of the "escapism" of some from their duties under the garb of spirituality and of the superficial modern educational varnish of the others, divorced from spiritual and moral standards. He, therefore, always laid great emphasis on the necessity of harmonising the 'spiritual' and the 'material' spheres of daily life. He also wanted to remove the false ideas, on the one hand, of those persons who think that Dharma can be practised by exclusively individual spiritual Sāadhanā coupled with more honest bread-earning, ignoring one's responsibility for rendering selfless service to the society and on the other hand of those who think that the Sāadhanā can be complete by mere service of society even without learning or practising any spirituality oneself. He wanted a happy blending of both. He stood for the omnilateral and all-round progress simultenaously of both the individual and society towards the speedy realisation of India's spiritual and cultural ideal, the lofty Vedantic ideal of 'Pūrṇatva' (perfection and harmony all-round).

With these ideas agitating his mind for several decades he went on carrying on a laborious, elaborate, patient and day-and-night research to evolve finally a splendid and perfect scheme for all-round reconstruction first of India and through it of the world. Consequently Sri Jagadguruji founded in 1953 at Nagpur an institution named Sri Vishwa Punarnirmana Sangha (World Reconstruction Association). The Administrative Board of the Sangha consisted of Jagadguruji's disciples, devotees and admirers of his idealistic and spiritual ideals for humanitarian service and included a number of high court judges, ministers, educationists, statesmen and other personage of the highest calibre

in Indian public life. It was, however, after a long and incessant search that Guruji had found his General Secretary Sri Chimanlal Trivedi whom he called his *Scipio Africanus* and who truly thought, worked, planned and dreamt unceasingly for the Sangha's welfare and progress. Although this Sangha could not function very effectively in the beginning on account of Jagadguruji's failing health, various pre-occupations and other unforeseen hurdles, it is actively engaged now in disseminating Jagadguruji's message and teachings with Justice B. P. Sinha, the Chief Justice of India as its President and Dr. C. D. Deshmukh, (I.C.S.) the ex-Finance Minister of India and ex-Chairman, University Grants Commission as its Vice-President.

With a view to promote the cause of world peace and to spread the lofty Vedantic spiritual ideals even outside India Shri Jagadguru went on a tour to America in February, 1958, the first tour outside India by a Shankaracharya in the history of the said Order. The tour was sponsored by Self Realisation Fellowship of Los-Angeles, the Vedantic Society founded by Paramhansa Yoganandji in America. Jagadguruji stayed there for about three months and during this period addressed rapt audiences in hundreds of colleges of universities, churches and other public institutions. He was also invited to give talks and mathematical demonstrations on the television. In fact, he released an exceptionally powerful current of moral and spiritual enlightenment, peace and harmony throughout America during his tour which proved a phenomenal success comparable perhaps with that of Swami Vivekananda only. A request was also received by him from Dr. Hornday, the Minister for Church of Religious Science to open a branch of Sri Vishwa Punarnirmana Sangha in America with a view to establish one religion all over the world. The suggestion, however, could not materialise at that time for certain reasons. On his way back Jagadguruji gave some lectures in U. K. also and returned to India in May, 1958.

Guruji had been undergoing a terrific strain for more than five decades in devoting his body, mind, heart and soul

to the cause of service of humanity, spreading of spiritual enlightenment and revival of Vedantic ideals. This had already undermined his health but still Gururji never devoted any attention to his personal comforts. The excessive strain of the vast hurricane tour abroad came as a severe blow to his health but still he refused to take rest and incessantly continued to pursue his studies, talks, lectures and writings with unabated and youthlike vigour and enthusiasm. In fact it required a great vigilance and heroic effort to prevent him from giving 'darshan', advice and talks to his devotees and disciples even when he could hardly speak on account of strain. As a result he fell seriously ill in Novembr, 1959 and despite the best available treatment shed off his mortal frame and took Mahasamadhi at Bombay on Feb. 2nd 1960.

From the very day of his assuming the throne of Jagadguru Shankaracharya, Sri Bharati Krishana Tirthaji had become the cynosure of all eyes. His winning personality, his charming innocence, his eager thirst for knowledge, his religious zeal, his earnest belief in the "sastras", his universal kindness, his retentive memory, all these attracted towards him every living soul that came in contact with him. People flocked to him in crowds and waited at his doors for hours together just to get a glimpse of that divine countenance. It was nothing but the divine lustre that shone in his face. It was nothing but the marvellous superhuman milk of kindness that flowed from his heart.

He was always perfectly impartial. Every one was equal in his eyes. He cared not for riches. He cared not for position. Nothing but Bhakti could attract people to him, rich or poor, high or low, everybody had to go through the portals of Bhakti to approach his august presence. Exhibiting his divinity, he loved as himself every one that came to him. Every one who had even two minute's conversation with him went out with the full conviction that he was the object of some special love of His Holiness.

Of such a divine personality it is impossible to draw a sketch. His activities were many-sided. To hear him was a

pleasure. To see him was a privilege. To speak to him was a real blessing and to be granted a special interview—Ah ! that was the acme of happiness which people coveted most in all earnestness. The magnetic force of his wonderful personality was such that one word, one smile, or even one look was quite enough to convert even the most sceptic into his most ardent and obedient disciple. He belonged to all irrespective of caste or creed and he was a real Guru to the whole world.

People of all nationalities, religions and climes, Brahmins and non-Brahmins, Hindus and Mahomedans, Parsis and Christians, Europeans and Americans received equal treatment at the hands of Mis Holiness. That was the secret of the immense popularity of this great Mahatma.

He was grand in his simplicity. People would give anything and everything to get his blessings and he would talk words of wisdom as freely without fear or favour. He was most easily accessible to all. Thousands of people visited him and prayed for the relief of their miseries. He had a kind word to say to each, after attentively listening to his or her tale of woe and then give them some 'prasad' which would cure their malady whether physical or mental. He would actually shed tears when he found people suffering and would pray to God to relieve their suffering.

He was mighty in his learning and voracious in his reading. A sharp intellect, a retentive memory and a keen zest went to mark him as the most distinguished scholar of his day. His leisure moments he would never spend in vain. He was always reading something or repeating something. There was no branch of knowledge which he did not know and that also 'shastrically'. He was equally learned in Chandahsastra, Ayurveda and Jyotish Sastra. He was a poet of uncommon merit and wrote a number of poems in Sanskrit in the praise of his guru, gods and goddesses with a charming flow of Bhakti so conspicuous in all his writings.

I have got a collection of over three thousand slokas forming part of the various eulogistic poems composed by Gurudeva

in adoration of various Devas and Devis. These Slokas have been edited and are being translated into Hindi. They are proposed to be published in three volumes along with Hindi translation.

The book on "Sanatana Dharma" by H. H. Swami Bhārati Kṛṣṇa Tirtha Mahārāja has been published by Bharatiya Vidya Bhavan, Bombay.

Above all, his Bhakti towards his Vidyāguru was something beyond description. He would talk for days together about the greatness of his Vidyāguru. He would be never tired of worshipping the Guru. His Guru also was equally attached to him and called our Swamiji as the own son of the Goddess of Learning, Shri Sarada. Everyday he would first worship his guru's sandals. His "Gurupādukā Stotra" clearly indicates the qualities he attributed to the sandals of his guru.

Shri Bhārati Kṛṣṇa Tirtha was a great Yogin and a "Siddha" of a very high order. Nothing was impossible for him. Above all he was a true Samnyasin. He held the world but as a stage where every one had to play a part. In short, he was undoubtedly a very great Mahātmā but without any display of mysteries or occultisms.

I have not been able to express here even one millionth part of what I feel. His spotless holiness, his deep piety, his endless wisdom, his childlike peacefulness, sportiveness and innocence and his universal affection beggar all description. His Holiness has left us a noble example of simplest living and highest thinking. May all the world benefit by the example of a life so nobly and so simply, so spiritually and so lovingly lived.

Introductory Remarks on the Present Volume

I now proceed to give a short account of the genesis of the work published herewith. Revered Guruji used to say that he had reconstructed the sixteen mathematical formulae (given in this text) from the Atharvaveda after assiduous research and 'Tapas' for about eight years in the

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forests surrounding Sringeri. Obviously these formulae are not to be found in the present recensions of Atharvaveda ; they were actually reconstructed, on the basis of intuitive revelation, from materials scattered here and there in the Atharvaveda. Revered Gurudeva used to say that he had written sixteen volumes (one for each Sūtra) on these Sūtras and that the manuscripts of the said volumes were deposited at the house of one of his disciples. Unfortunately, the said manuscripts were lost irretrievably from the place of their deposit and this colossal loss was finally confirmed in 1956. Revered Gurudeva was not much perturbed over this irretrievable loss and used to say that everything was there in his memory and that he could re-write the 16 volumes !

My late husband Sri C. M. Trivedi, Hon. Gen. Secertary V. P. Sangh noticed that while Sri Jagadguru Maharaj was busy demonstrating before learned people and societies Vedic Mathematics as discovered and propounded by him, some persons who had grasped a smattering of the new Sūtras had already started to dazzle audiences as prodigies claiming occult powers without aknowledging indebtedness to the Sūtras of Jagadguruji. My husband, therefore, pleaded earnestly with Gurudeva and persuaded him to arrange for the publication of the Sūtras in his own name.

In 1957, when he had decided finally to undertake a tour of the U.S.A. he re-wrote from memory the present volume, giving an introductory account of the sixteen formulae reconstructed by him. This volume was written in his old age within one month and a half with his failing health and weak eyesight. He had planned to write subsequent volumes, but his failing health (and cataract developed in both eyes) did not allow the fulfilment of his plans. Now the present volume is the only work on Mathematics that has been left over by Revered Guruji ; all his other writings on Vedic Mathematics have, alas, been lost for ever.

The typescript of the present volume was left over by Revered Gurudeva in U.S.A. in 1958 for publication. He

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had been given to understand that he would have to go to the U.S.A. for correction of proofs and personal supervision of printing. But his health deteriorated after his return to India and finally the typescript was brought back from the U.S.A. after his attainment of Mahasamadhi, in 1960.

ACKNOWLEDGEMENTS

I owe a deep debt of gratitude to Justice N. H. Bhagwati, the enlightened Vice-Chancellor of the Banaras Hindu University and other authorities of the B.H.U. who have readily undertaken the publication of this work which was introduced to them by Dr. Pt. Omkarnath Thakur. I am indebted to Dr. Thakur for this introduction. My hearty and reverent thanks are due to Dr. V. S. Agrawala (Professor, Art & Architecture, B.H.U.) the veteran scholar, who took the initiative and throughout kept up a very keen interest in this publication. It is my pleasant duty to offer my heartfelt gratitude to Dr. Prem Lata Sharma, Dean, Faculty of Music and Fine Arts, B.H.U. who voluntarily took over the work of press-dressing of the typescript and proof-reading of this volume after a deadlock had come to prevail in the process of printing just at the outset. But for her hard labour which she has undertaken out of a sheer sense of reverence for the noble and glorious work of Revered Gurudeva this volume would not have seen the light of the day for a long time. I trust that Revered Gurudeva's Holy Spirit will shower His choicest blessings on her. My sincere thanks are also due to Sri S. Nijabodha of the Research Section under the charge of Dr. Sharma, who has ably assisted her in this onerous task.

The Humblest of His Disciples

Smti. MANJULA TRIVEDI

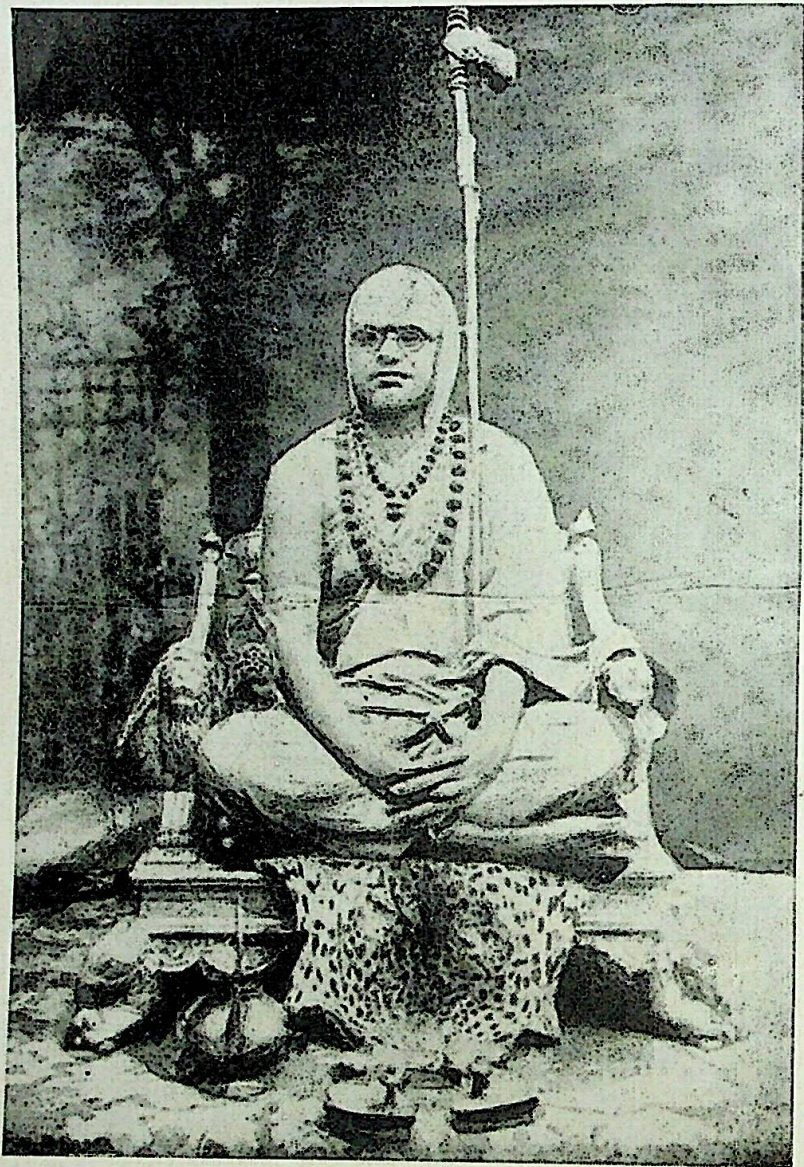
Hony. General Secretary

SRI Vishwa Punarnirmana
Sangha, Nagpur.

Nagpur,

16th March, 1965.





The Author
Jagadguru Śaṅkarācārya
Śrī Bhāratī Kṛṣṇa Tīrtha Mahārāja
(1884—1960)

AUTHOR'S PREFACE

A.—A DESCRIPTIVE PREFATORY NOTE

ON

THE ASTOUNDING WONDERS

OF

ANCIENT INDIAN VEDIC MATHEMATICS

1. In the course of our discourses on manifold and multifarious subjects (spiritual, metaphysical, philosophical, psychic, psychological, ethical, educational, scientific, mathematical, historical, political, economic, social etc., etc., from time to time and from place to place during the last five decades and more, we have been repeatedly pointing out that the Vedas (the most ancient Indian scriptures, nay, the oldest "Religious" scriptures of the whole world) claim to deal with all branches of learning (spiritual and temporal) and to give the earnest seeker after knowledge all the requisite instructions and guidance in full detail and on scientifically—nay, mathematically—accurate lines in them all and so on.

2. The very word "Veda" has this derivational meaning i.e. the fountain-head and illimitable store-house of all knowledge. This derivation, in effect, means, connotes and implies that the Vedas should contain within themselves all the knowledge needed by mankind relating not only to the so-called 'spiritual' (or other-worldly) matters but also to those usually described as purely "secular", "temporal", or "wordly"; and also to the means required by humanity as such for the achievement of all-round, complete and perfect success in all conceivable directions and that there can be no adjectival or restrictive epithet calculated (or tending) to limit that knowledge down in any sphere, any direction or any respect whatsoever.

3. In other words, it connotes and implies that our ancient Indian Vedic lore should be all-round complete and perfect and able to throw the fullest necessary light on all matters which any aspiring seeker after knowledge can possibly seek to be enlightened on.

4. It is thus in the fitness of things that the Vedas include (i) *Āyurveda* (anatomy, physiology, hygiene, sanitary science, medical science, surgery etc., etc.) not for the purpose of achieving perfect health and strength in the after-death future but in order to attain them *here and now* in our present physical bodies; (ii) *Dhanurveda* (archery and other military sciences) not for fighting with one another after our transportation to heaven but in order to quell and subdue all invaders from abroad and all insurgents from within; (iii) *Gāndharva Veda* (the science and art of music) and (iv) *Sihāpatya Veda* (engineering, architecture etc., and all branches of mathematics in general). All these subjects, be it noted, are inherent parts of the Vedas i.e. are reckoned as "spiritual" studies and catered for as such therein.

5. Similar is the case with regard to the *Vedāṅgas* (i.e. grammar, prosody, astronomy, lexicography etc., etc.) which, according to the Indian cultural conceptions, are also inherent parts and subjects of *Vedic* (i.e. *Religious*) study.

6. As a direct and unshirkable consequence of this analytical and grammatical study of the real connotation and full implications of the word "Veda" and owing to various other historical causes of a personal character (into details of which we need not now enter), we have been from our very early childhood, most earnestly and actively striving to study the Vedas critically from this stand-point and to realise and prove to ourselves (and to others) the correctness (or otherwise) of the derivative meaning in question.

7. There were, too, certain personal historical reasons why in our quest for the discovering of all learning in all its departments, branches, sub-branches etc., in the Vedas, our gaze was riveted mainly on ethics, psychology and metaphysics on the one hand and on the "positive" sciences and especially mathematics on the other.

8. And the contemptuous or, at best patronising attitude adopted by some so-called Orientalists, Indologists, anti-quarians, research-scholars etc., who condemned, or light-

heartedly, nay ; irresponsibly, frivolously and flippantly dismissed, several abstruse-looking and recondite parts of the Vedas as "sheer-nonsense"—or as "infant-humanity's prattle", and so on, merely added fuel to the fire (so to speak) and further confirmed and strengthened our resolute determination to unravel the too-long hidden mysteries of philosophy and science contained in ancient India's Vedic lore, with the consequence that, after eight years of concentrated contemplation in forest-solitude, we were at long last able to recover the long lost keys which alone could unlock the portals thereof.

9. And we were agreeably astonished and intensely gratified to find that exceedingly tough mathematical problems (which the mathematically most advanced present day Western scientific world had spent huge lots of time, energy and money on and which even now it solves with the utmost difficulty and after vast labour involving large numbers of difficult, tedious and cumbersome "steps" of working) can be easily and readily solved with the help of these ultra-easy Vedic Sūtras (or mathematical aphorisms) contained in the *Parīṣiṣṭa* (the Appendix-portion) of the *ĀTHARVAVEDA* in a few simple steps and by methods which can be conscientiously described as mere "mental arithmetic".

10. Eversince (i.e. since several decades ago), we have been carrying on an incessant and strenuous campaign for the India-wide diffusion of all this scientific knowledge, by means of lectures, blackboard- demonstrations, regular classes and so on in schools, colleges, universities etc., all over the country and have been astounding our audiences everywhere with the wonders and marvels not to say, miracles of Indian Vedic mathematics.

11. We were thus at last enabled to succeed in attracting the more than passing attention of the authorities of several Indian universities to this subject. And, in 1952, the Nagpur University not merely had a few lectures and blackboard-demonstrations given but also arranged for our holding regular classes in Vedic mathematics (in the University's Convocation

Hall) for the benefit of all in general and especially of the University and college professors of mathematics, physics etc.

12. And, consequently, the educationists and the cream of the English educated section of the people including the highest officials (e.g. the high-court judges, the ministers etc.) and the general public as such were all highly impressed; nay, thrilled, wonder-struck and flabbergasted! And not only the newspapers but even the University's official reports described the tremendous sensation caused thereby in superlatively eulogistic terms; and the papers began to refer to us as "the Octogenarian Jagadguru Shankaracharya who had taken Nagpur by storm with his Vedic mathematics", and so on!

13. It is manifestly impossible, in the course of a short note (in the nature of a "trailer"), to give a full, detailed, thorough-going, comprehensive and exhaustive description of the unique features and startling characteristics of all the mathematical lore in question. This can and will be done in the subsequent volumes of this series (dealing seriatim and in extenso with all the various portions of all the various branches of mathematics).

14. We may, however, at this point, draw the earnest attention of every one concerned to the following salient items thereof:—

- (i) The Sūtras (aphorisms) apply to and cover each and every part of each and every chapter of each and every branch of mathematics (including arithmetic, algebra, geometry—plane and solid, trigonometry—plane and spherical, conics—geometrical and analytical, astronomy, calculus—differential and integral etc., etc. In fact, there is no part of mathematics, pure or applied, which is beyond their jurisdiction;
- (ii) The Sūtras are easy to understand, easy to apply and easy to remember; and the whole work can be truthfully summarised in one word "mental"!

- (iii) Even as regards complex problems involving a good number of mathematical operations (consecutively or even simultaneously to be performed), the time taken by the Vedic method will be a third, a fourth, a tenth or even a much smaller fraction of the time required according to modern (i.e. current) Western methods ;
- (iv) And, in some very important and striking cases, sums requiring 30, 50, 100 or even more numerous and cumbrous "steps" of working (according to the current Western methods) can be answered in a single and simple step of work by the Vedic method ! And little children (of only 10 or 12 years of age) merely look at the sums written on the blackboard (on the platform) and immediately shout out and dictate the answers from the body of the convocation hall (or other venue of the demonstration). And this is because, as a matter of fact, each digit automatically yields its predecessor and its successor ! and the children have merely to go on tossing off (or reeling off) the digits one after another (forwards or backwards) by mere mental arithmetic (without needing pen or pencil, paper or slate etc) !
- (v) On seeing this kind of work actually being performed by the little children, the doctors, professors and other "big-guns" of mathematics are wonder struck and exclaim :—"Is this mathematics or magic" ? And we invariably answer and say : "It is both. It is magic until you understand it ; and it is mathematics thereafter" ; and then we proceed to substantiate and prove the correctness of this reply of ours ! And
- (vi) As regards the time required by the students for mastering the whole course of Vedic mathematics as applied to all its branches, we need merely state from our actual experience that 8 months (or 12 months) at an average rate of 2 or 3 hours per day

should suffice for completing the whole course of mathematical studies on these Vedic lines instead of 15 or 20 years required according to the existing systems of the Indian and also of foreign universities.

15. In this connection, it is a gratifying fact that unlike some so-called Indologists (of the type hereinabove referred to) there have been some great modern mathematicians and historians of mathematics (like Prof. G. P. Halstead, Professor Ginsburg, Prof. De Morgan, Prof. Hutton etc.,) who have, as truth-seekers and truth-lovers, evinced a truly scientific attitude and frankly expressed their intense and whole-hearted appreciation of ancient India's grand and glorious contributions to the progress of mathematical knowledge (in the Western hemisphere and elsewhere).

16. The following few excerpts from the published writings of some universally acknowledged authorities in the domain of the history of mathematics, will speak eloquently for themselves :—

- (i) On page 20 of his book "On the Foundation and Technique of Arithmetic", we find Prof. G.P. Halstead saying "The importance of the creation of the ZERO mark can never be exaggerated. This giving of airy nothing not merely a local habitation and a name, a picture but helpful power is the characteristic of the Hindu race whence it sprang. It is like coining the Nirvāṇa into dynamos. No single mathematical creation has been more potent for the general on-go of intelligence and power".
- (ii) In this connection, in his splendid treatise on "The present mode of expressing numbers" (the Indian Historical Quarterly Vol. 3, pages 530-540) B. B. Dutta says: "The Hindus adopted the decimal scale very early. The numerical language of no other nation is so scientific and has attained as high a state of perfection as that of the ancient Hindus.

In symbolism they succeeded with ten signs to express any number most elegantly and simply. It is this beauty of the Hindu numerical notation which attracted the attention of all the civilised peoples of the world and charmed them to adopt it”.

(iii) In this very context, Prof. Ginsburg says :—

“The Hindu notation was carried to Arabia about 770 A.D. by a Hindu scholar named KAN̄KA who was invited from Ujjain to the famous Court of Baghdad by the Abbaside Khalif AL-MANSUR. Kan̄ka taught Hindu astronomy and mathematics to the Arabian scholars ; and, with his help, they translated into Arabic the Brahma-Sphuṭa-Siddhānta of Brahma Gupta. The recent discovery by the French savant M.F. NAU proves that the Hindu numerals were well known and much appreciated in Syria about the middle of the 7th Century A-D”. (GINSBURG’S “NEW LIGHT on our numerals”, Bulletin of the American Mathematical Society, Second series, Vol. 25, pages 366-369).

(iv) On this point, we find B. B. Dutta further saying :

“From Arabia, the numerals slowly marched towards the West through Egypt and Northern Arabia ; and they finally entered Europe in the 11th Century. The Europeans called them the Arabic notations, because they received them from the Arabs. But the Arabs themselves, the Eastern as well as the Western, have unanimously called them the Hindu figures. (Al-Arqan-Al-Hindu”).

17. The above-cited passages are, however, in connection with and in appreciation of India’s invention of the “ZERO” mark and her contributions of the 7th century A.D. and later to world mathematical knowledge.

In the light, however, of the hereinabove given detailed description of the unique merits and characteristic excellences of the still earlier Vedic Sūtras dealt with in the 16 volumes of

this series¹, the conscientious (truth-loving and truth-telling) historians of Mathematics (of the lofty eminence of Prof. De Morgan etc.) have not been guilty of even the least exaggeration in their candid admission that “even the highest and farthest reaches of modern Western mathematics have not yet brought the Western world even to the threshold of Ancient Indian Vedic Mathematics”.

18. It is our earnest aim and aspiration, in these 16 volumes¹, to explain and expound the contents of the Vedic mathematical Sūtras and bring them within the easy intellectual reach of every seeker after mathematical knowledge.

B.—EXPLANATORY EXPOSITION
OF
SOME SALIENT, INSTRUCTIVE AND
INTERESTING ILLUSTRATIVE SAMPLE SPECIMENS
BY WAY OF
COMPARISON AND CONTRAST

Preliminary Note :—

With regard to every subject dealt with in the Vedic Mathematical Sūtras, the rule generally holds good that the Sūtras have always provided for what may be termed the ‘General Case’ (by means of simple processes which can be easily and readily—nay, instantaneously applied to any and every question which can possibly arise under any particular heading.

2. But, at the same time, we often come across special cases which, although classifiable under the general heading in question, yet present certain additional and typical characteristics which render them still easier to solve. And, therefore, special provision is found to have been made for such special cases by means of special Sūtras, sub-Sūtras, corollaries etc., relating and applicable to those particular types alone.

¹ Only one volume has been bequeathed by His Holiness to posterity of p. x above—*General Editor*.

3. And all that the student of these Sūtras has to do is to look for the special characteristics in question, recognise the particular type before him and determine and apply the special formula prescribed therefor.

4. And, generally speaking it is only *in case* no special case is involved, that the general formula has to be resorted to. And this process is naturally a little longer. But it need hardly be pointed out that, even then, the longest of the methods according to the Vedic system comes nowhere (in respect of length, cumbrousness and tediousness etc.,) near the corresponding process according to the system now current everywhere.

5. For instance, the conversion of a vulgar fraction (say $\frac{1}{18}$ or $\frac{1}{28}$ or $\frac{1}{42}$ etc.,) to its equivalent recurring decimal shape involves 18 or 28 or 42 or more steps of cumbrous working (according to the current system) but requires only one single and simple step of mental working (according to the Vedic Sūtras)!

6. This is not all. There are still other methods and processes (in the latter system) whereby even that very small (mental) working can be rendered shorter still! This and herein is the beatific beauty of the whole scheme.

7. To start with, we should naturally have liked to begin this explanatory and illustrative exposition with a few processes in arithmetical computations relating to multiplications and divisions of huge numbers by big multipliers and big divisors respectively and then go on to other branches of mathematical calculation.

8. But, as we have *just hereinabove* referred to a particular but wonderful type of mathematical work wherein 18, 28, 42 or even more steps of working can be condensed into a single-step answer which can be written down immediately (by means of what we have been describing as straight, single-line, mental arithmetic); and, as this statement must naturally have aroused intense eagerness and curiosity in the minds of the students (and the teachers too) and especially as the process is

based on elementary and basic fundamental principles and requires no previous knowledge of anything in the nature of an indispensable and inescapable pre-requisite chapter, subject and so on, we are beginning this exposition here with an easy explanation and a simple elucidation of that particular illustrative specimen.

9. And then we shall take up the other various parts, one by one, of the various branches of mathematical computation and hope to throw sufficient light thereon to enable the students to make their own comparison and contrast and arrive at correct conclusions on all the various points dealt with.

C. ILLUSTRATIVE SPECIMEN SAMPLES

(*Comparison and Contrast*)

SAMPLE SPECIMENS

OF

ARITHMETICAL COMPUTATIONS

I. *Multiplication.*: The "Sanskrit Sūtra" (Formula) is—

(i) *Multiply 87265 by 32117* ॥ ऊर्ध्वंतिर्यग्भ्याम् ॥

By current method : *By Vedic mental one-line method :*

87265
32117

610855
87265
87265
174530
261795

2802690005

87265
32117

2802690005

Note : Only the answer is written automatically down by *Urdhwa Tiryak Sūtra* (forwards or backwards).

(xxiii)

II. Division :

(2) Express $\frac{1}{19}$ in its full recurring decimal shape (18 digits) :

By the current method : The "Sanskrit Sūtra" (Formula) is ;

19) 1.00(.052631578947368421 ॥ एकाधिकेन पूर्वेण ॥

95

By the Vedic mental one-line method :

50

(by the Ekādhika-Pūrva Sūtra)

38

(forwards or backwards), we merely

120

write down the 18-digit-answer :—

114

.052631578 } i.
947368421 }

60

57

30

19

110

95

150

133

170

152

180

171

90

76

140

133

70

57

130

114

160

152

80

76

40

38

20

19

1

Division continued :

Note : $\frac{1}{48}$ gives 42 recurring decimal places in the answer but these too are written down mechanically in the same way (backwards or forwards). And the same is the case with all such divisions (whatever the number of digits may be):

(3) *Divide 7031985 by 823 :*

By the current method : *By the mental Vedic one-line method :*

$ \begin{array}{r} 823 \overline{) 7031985(8544} \\ \underline{6584} \\ 4479 \\ \underline{4115} \\ 3648 \\ \underline{3292} \\ 3565 \\ \underline{3292} \\ 273 \end{array} $	$ \begin{array}{r} 8123 \overline{) 70319(85} \\ \underline{675} \\ 8544(273 \end{array} $
--	---

$$\therefore Q = 8544$$

$$R = 273$$

(4) *Divide .0003147 by 814256321 (to 6 decimal places) :*

The current method is notoriously too long, tedious, cumbersome and clumsy and entails the expenditure of enormous time and toil. Only the Vedic mental one-line method is given here. The truth-loving student can work it out by the other method and compare the two for himself.

$$\begin{array}{r}
 8/1425632) \cdot 0003147 \\
 3295 \\
 \hline
 0000419...
 \end{array}$$

(5) *Find the Reciprocal of 7246041 to eleven Decimal places :*

By the Vedic mental one-line-method.

(by the *Ūrdhwa-Tiryak Sūtra*)

$$\begin{array}{r}
 7/246041) \cdot 000001000000 \\
 374610 \\
 \hline
 00000013800...
 \end{array}$$

*N.B. :—*The same method can be used for 200 or more places :

III. *Divisibility* :(6) *Find out whether 5293240096 is divisible by 139 :*

By the current method, nothing less than complete division will give a clue to the answer (Yes or No).

But by the Vedic mental one-line method (by the *Ekādhika-Pūrva Sūtra*), we can at once say :—

for) 5 2 9 3 2 4 0 0 9 6 } ∴ YES.
139) 139 89 36 131 29 131 19 51 93

IV. *Square Root* :(7) *Extract the square root of 738915489 :*

By the current method : *By the Vedic mental one-line method :*

$ \begin{array}{r} 738915489(27183 \\ \underline{4} \\ 47)338 \\ \underline{329} \\ 541) 991 \\ \underline{541} \\ 5428) 45054 \\ \underline{43424} \\ 54363) 163089 \\ \underline{163089} \\ \underline{0} \end{array} $	$ \begin{array}{r} 4)738915489 \\ \underline{35513674} \\ 27183.000 \text{ Ans.} \end{array} $ <p>(By the <i>Ūrdhwa-Tiryak Sūtra</i>)</p>
---	---

∴ The square root is 27183.

(8) *Extract the square root of 19·706412814 to 6 decimal places :*

The current method is too cumbrous and may be tried by the student himself.

The Vedic mental one-line method (by *Ūrdhwa-Tiryak Sūtra*) is as follows :—

$$\begin{array}{r}
 8)19\cdot706412814 \\
 \underline{\cdot 351010151713} \\
 4\cdot439190 \dots
 \end{array}$$

V. *Cubing and Cube-Root :*

The "Sanskrit Sūtra"
(Formula) is :—

(9) *Find the cube of 9989.*

॥ यावद्गुणं तावद्गुणीकृत्य वर्गं च योजयेत् ॥

The current method is too cumbrous.

The Vedic mental one-line method (by the *Yāvadūnam-Tāvadūnam Sūtra*) is as follows :—

$$9989^3 = 9967/0363/1331 = 9967/0362/8669$$

(10) *Extract the Cube-Root of 355045312441 :*

The current method is too cumbrous.

The Vedic mental one-line method is as follows :—

$$\sqrt[3]{355045312441} = 7 \dots 1 = 7081$$

SAMPLE SPECIMENS FROM ALGEBRA

I. *Sample Equations :*

The "Sanskrit Sūtra"
(Formula) is :—

(11) Solve : $\frac{3x+4}{6x+7} = \frac{x+1}{2x+3}$

॥ शून्यं साम्यसमुच्चये ॥

By the current method :

By the Vedic method (by the *Sūnyam-Samuccaya Sūtra*)

$$\therefore 6x^2 + 17x + 12 =$$

$$\therefore 4x + 5 = 0 \therefore x = -1\frac{1}{4}$$

$$6x^2 + 13x + 7$$

$$\therefore 4x = -5$$

$$\therefore x = -1\frac{1}{4}$$

(12) $\frac{4x+21}{x+5} + \frac{5x-69}{x-14} = \frac{3x-5}{x-2} + \frac{6x-41}{x-7}$

The current method is too cumbrous.

The Vedic method simply says : $2x-9=0 \therefore x=4\frac{1}{2}$ (13) $\left(\frac{x-5}{x-7}\right)^3 = \frac{x-3}{x-9}$

The current method is horribly cumbrous.

The Vedic method simply says : $4x-24=0 \therefore x=6$.II. *Quadratic-Equations (and Calculus) :*

The same is the case here.

$$(14) \frac{16x-3}{7x+7} = \frac{2x-15}{11x-25} \therefore x=1 \text{ or } 10/9$$

$$(15) \frac{3}{x+3} + \frac{4}{x+4} = \frac{2}{x+2} + \frac{5}{x+5} \therefore x=0 \text{ or } -7/2.$$

$$(16) 7x^2 - 11x - 7 = 0$$

By Vedic method (by "*Calana-kalana*" Sūtra ; Formula)
i.e., by Calculus-Formula we say: $14x-11 = \pm \sqrt{317}$.

N.B. :—Every quadratic can thus be broken down into two binomial-factors. And the same principle can be utilised for cubic, biquadratic, pentic etc., expressions.

III. *Summation of Series :*

The current methods are horribly cumbrous. The Vedic mental one-line methods are very simple and easy.

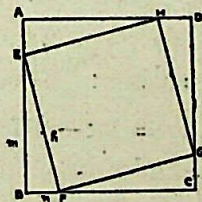
$$(17) \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110} = 4/77$$

$$(18) \frac{3}{70} + \frac{9}{190} + \frac{27}{374} + \frac{81}{6870} = \frac{133}{1015}$$

SPECIMEN SAMPLES FROM GEOMETRY

(19) Pythagoras Theorem is constantly required in all mathematical work, but the proof of it is ultra-notorious for its cumbrousness, clumsiness, etc. There are several Vedic proofs thereof (every one of them much simpler than *Euclid's*). I give two of them below:—

E, F, G and H are points on AB, BC, CD and DA such that AE=BF=CG=DH. Thus ABCD is split up into the square EFGH and 4 congruent triangles.



$$\begin{aligned} \text{Their total area} &= H^2 + 4x\frac{1}{2} \times mn \\ &\rightarrow (H^2 + 4x\frac{1}{2} mn) \\ &= H^2 + 2mn \end{aligned}$$

$$\begin{aligned} \text{But the area of ABCD is } &(m+n)^2 \\ &= m^2 + 2mn + n^2 \end{aligned}$$

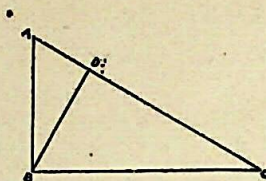
$$\therefore H^2 + 2mn = m^2 + 2mn + n^2$$

$$\therefore H^2 = m^2 + n^2. \text{ Q.E.D.}$$

(20) *Second Proof :*

Draw $BD \perp$ to AC .

Then ABC , ADB and BDC are similar.



$$\therefore \frac{ADB}{ABC} = \frac{AB^2}{AC^2} \text{ and } \frac{BDC}{ABC} = \frac{BC^2}{AC^2}$$

$$\therefore \frac{ADB+BDC}{ABC} = \frac{AB^2+BC^2}{AC^2}. \text{ But ; } ADB+BDC=ABC$$

$$\therefore AB^2+BC^2=AC^2. \text{ Q.E.D.}$$

Note :—Apollonius Theorem, Ptolemy's Theorem, etc., etc., are all similarly proved by very simple and easy methods.

SIMPLE SPECIMENS FROM CONICS AND CALCULUS

(21) *Equation of the straight line joining two points :*

For finding the equation of the straight line passing through two points (whose co-ordinates are given).

Say $(9, 17)$ and $(7, -2)$.

By the Current Method :

Let the equation be $y = mx + c$.

$$\therefore 9m + c = 17; \text{ and } 7m + c = -2$$

Solving this simultaneous-equation in m and c .

$$\text{We have } 2m = 19; \therefore m = 9\frac{1}{2} \therefore C = -68\frac{1}{2}$$

Substituting ; these, values, we have $y = 9\frac{1}{2}x - 68\frac{1}{2}$

$$\therefore 2y = 19x - 137 \therefore 19x - 2y = 137. \text{ But this method is cumbrous.}$$

Second method using the formula $y - y^1 = \frac{y^{11} - y^1}{x^{11} - x^1} (x - x^1)$

is still more cumbrous (and confusing).

But the *Vedic mental one-line method* by the Sanskrit Sūtra (Formula), ॥ परावर्त्यं योजयेत् ॥ ("*Parāvartya-Sūtra*") enables us to write down the answer *by a mere look* at the given co-ordinates.

(22) When does a general-equation represent two straight lines ?

Say, $12x^2 + 7xy - 10y^2 + 13x + 45y - 35 = 0$

By the Current Method.

Prof. S. L. Loney devotes about 15 lines (section 119, Ex. 1 on page 97 of his "Elements of Co-ordinate Geometry") to his "model" solution of this problem as follows :—

Here $a=12$, $h=7/2$, $b=-10$, $g=13/2$, $f=45/2$ and $c=-35$.

$$\begin{aligned} \therefore abc + 2fgh - af^2 - bg^2 - ch^2 &\text{ turns out to be zero.} \\ = 12(x-10)(x-35) + \frac{2 \times 45}{2} \times \frac{13}{2} \times \frac{7}{2} - \frac{12(45)^2}{2} - (-10) \frac{(13)^2}{2} \\ - (-35) \frac{(7)^2}{2} &= 4200 + \frac{4095}{4} - 6075 + \frac{1690}{4} + \frac{1715}{4} = -1875 + \frac{7500}{4} = 0 \end{aligned}$$

The equation represents two straight lines.
Solving it for x , we have :—

$$\begin{aligned} x^2 + \frac{7y+13}{12} + \left(\frac{7y+13}{24}\right)^2 &= \frac{10y^2-45y+35}{12} + \left(\frac{7y+13}{24}\right)^2 \\ &= \left(\frac{23y-43}{24}\right)^2 \end{aligned}$$

$$\therefore x + \frac{7y+13}{24} = \frac{23y-43}{24}$$

$$\therefore x = \frac{2y-7}{3} \text{ or } -\frac{5y+5}{4}$$

\therefore The two straight lines are $3x=2y-7$ and $4x=-5y+5$.

By the Vedic method, however, we at once apply the "*Ādyam-ādyena*" Sūtra and (by merely looking at the quadratic) write down the answer :

Yes; and the straight lines are $3x-2y=-7$ and $4x+5y=5$.

(23) Dealing with the same principle and adopting the same procedure with regard to hyperbolas, conjugate hyperbolas and asymptotes, in articles 324 and 325 on pages 293 and 294 of his "Elements of Co-ordinate Geometry" Prof. S. L. Loney devotes $27+14(=41)$ lines) to the problem and says :—



(xxx)

As $3x^2 - 5xy - 2y^2 + 5x - 11y - 8 = 0$ is the equation to the given hyperbola.

$$\therefore 3(-2)c + 2 \cdot \frac{5}{2} \cdot \frac{11}{2} \left(-\frac{5}{2}\right) - 3\left(\frac{11}{2}\right)^2 - (-2)\left(\frac{5}{2}\right)^2 - c\left(-\frac{5}{2}\right)^2 = 0.$$

$$\therefore c = -12.$$

\therefore The equation to the asymptotes is $3x^2 - 5xy - 2y^2 + 5x - 11y - 12 = 0$

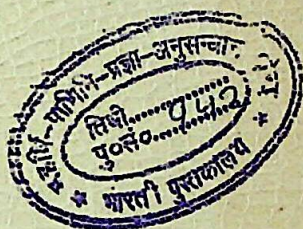
and the equation to the conjugate-hyperbola is $3x^2 - 5xy - 2y^2 + 5x + 15y - 16 = 0$

By the Vedic method, however, we use the same ('*Ādyam-ādyena*') Sūtra and automatically write down the equation to the asymptotes and the equation to the conjugate-hyperbola.

The Vedic methods are so simple that their very simplicity is astounding, and, as Desmond Doig has aptly, remarked, *it is difficult for any one to believe it until one actually sees it.*

It will be our aim in this and the succeeding volumes¹ to bring this long-hidden treasure-trove of mathematical knowledge within easy reach of everyone who wishes to obtain it and benefit by it.

¹ This is the only volume left by the author—Editor.



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